PROJECT RAND

DOCUMENT

SOME GAMES AND MACHINES FOR PLAYING THEM

John Nash

D-1164

2 February 1952

Summary: Some board games are presented which have the following properties:
1) The first player has a winning strategy.
2) The winning strategy is, as yet, unknown.
A description of electrical analog machines capable of playing these games are given. In practice, these machines play the games fairly well, but not perfectly.

SOME GAMES AND MACHINES FOR PLAYING THEM

John Nash

This report is stimulated by a request from J. D. Williams for examples of games and by a very interesting conversation with Claude Shannon.

Shannon and his group at Bell Telephone Laboratories have been working on the general problem of designing machines to play games, such as chess. The general objective is not to try to get a machine which will play perfectly but to obtain one that can play moderately well, so as to be able to compete with a reasonably intelligent human who does not have too much experience with the game or any detailed knowledge of the nature of the machine.

The first game is called "Hex." It seems that it was originally discovered in Denmark, and rediscovered by the author at Princeton.

Hex is played on a rhomboidal board which is subdivided into hexagons which fit together as in Figure 1. Each of the two players, white and black, has a pair of opposite sides of the rhombus and it is his objective to connect them by a chain of stones of his color.

The players move alternately; and each time one of them moves he puts a stone of his color on the board. The game ends when a player has succeeded in connecting his sides of the board by a chain of stones of his color. [See Fig.,1, where black has won.]

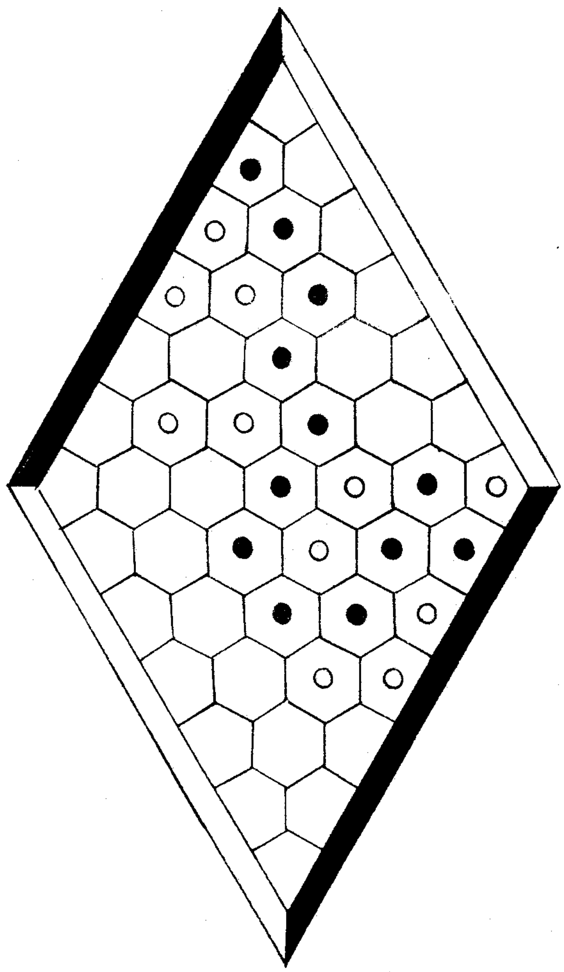

Fig. 1

The nature of the game is such that if the board were completely filled with stones either white would have connected or black would have done so. And connection and blocking the opponent are equivalent acts. In Hex it is never disadvantageous to have an extra stone on the board. This is quite unlike the situation in chess, or Go, where the possession of a piece, or stone, at a particular location on the board can be a real handicap.

From the properties of the game just mentioned above one can give a simple contradiction argument showing that the player who moves second cannot have a winning strategy and thus that the first player can always win if he plays properly. However, the first player does not seem to have a <u>simple</u> winning strategy on a large board game. It is possible, however, that he has a dualization type strategy. If such were the case he could guide his play by a division of the board into 3 classes of hexagons:

(a) His first move

(b) A class of paired hexagons

(c) A class of dummy hexagons.

When his opponent occupies one of a pair of hexagons he occupies the other, unless he already has done so, in which case he may play anywhere. The pattern must be such that any connection path for the opponent either contains two paired hexagons or the first move hexagon.

Shannon's group have devised an electrical analog machine to play Hex. Think of the hexagons as terminals of an electrical network. Adjacent ones are connected by a standard size resistor.

And each hexagon on the white (black) sides of the boards is connected to a certain plus (minus) electrical source +(-) through a resistor of the same type. Of course, the vertex terminals connect both to + and to −. [See Fig. 2]

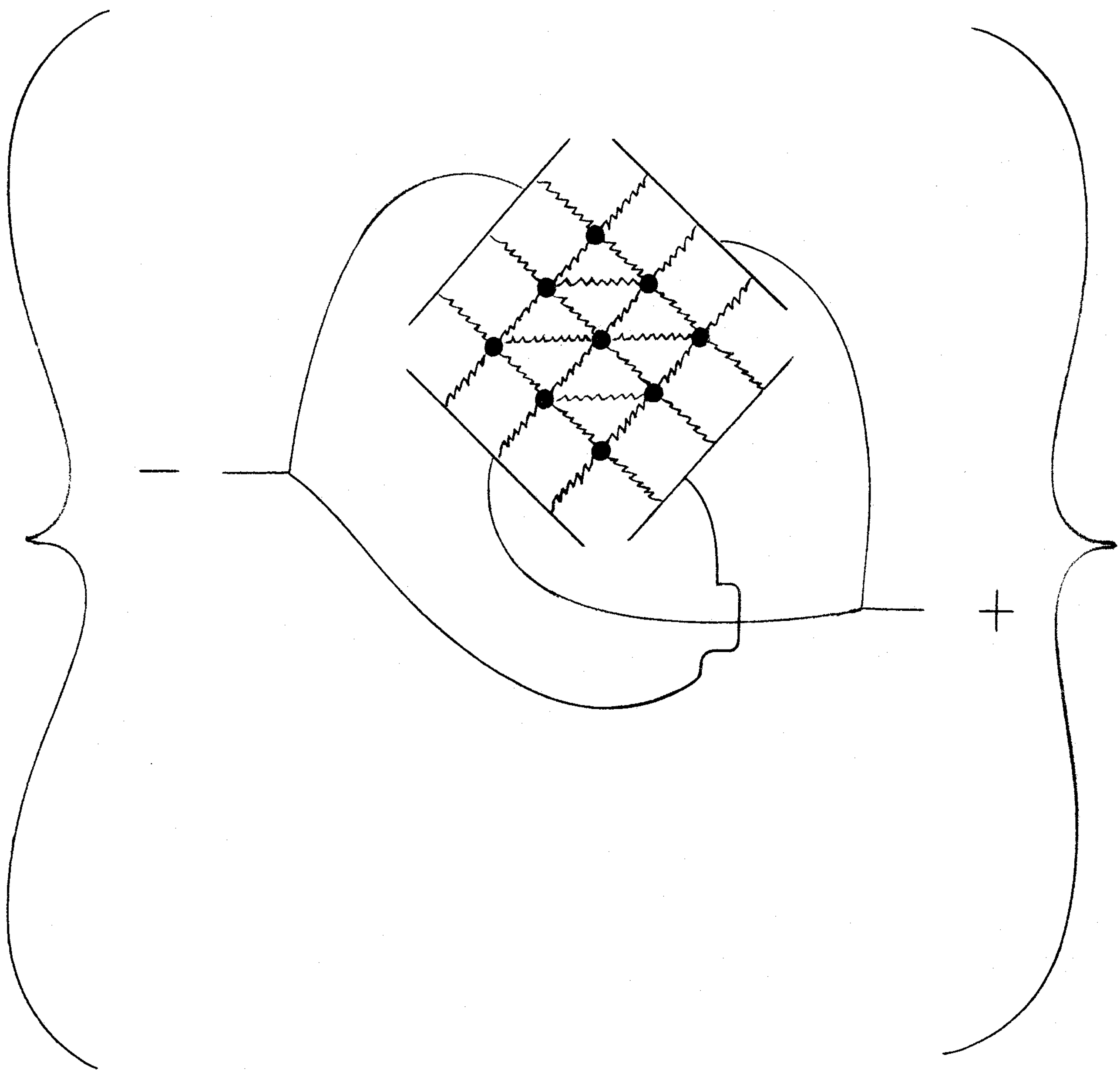

Fig. 2

When stones are placed on the board the network is modified by connecting a hexagon to + if occupied by a black stone. These are direct connections (zero resistance).

When his turn comes, the machine (let us assume the machine is black) chooses a move by finding a "path of least resistance" along with the maximum of the positive potential is least. The path must of course connect his two sides. Then he plays at a point on this path at maximum positive potential (aweakest link). Observe that this is a mini-max principle.

Shannon says that on the 7×7 board the machine plays a fair, but not perfect game.

Various modifications of Hex have been played at Princeton and at Bell Labs. There is the negative game, where you try not to connect. This is a rather slow game. The conjecture is that first player wins on even boards, second player on odd boards. This conjecture can be proved if one assumes first player has a winning dualization-type strategy. In "Tate" the first player starts with a single move and after that each player makes two moves at a time. It seems that the first player wins on odd boards, second on even boards.

Hex can be modified by using other sorts of boards. There is toroidal Hex and projective plane Hex. The main difficulty in playing these is in forming a clear picture of the situation. "Triangle" is a game of this type originally discovered by J. W. Milnor and rediscovered at Bell Labs. The hexagons are arranged in a large triangle and the objective of each player is to form a connected set of stones which touches all three sides.

If the board is completely filled with stones one, and only one, of the players will have accomplished this. The first player seems to have a rather large advantage.

The Bell Labs men have found a class of games based on self-dual graphs in which the first player always wins. For example, in the case illustrated [Fig. 3] player A wants to connect and B and player B wants to prevent this.

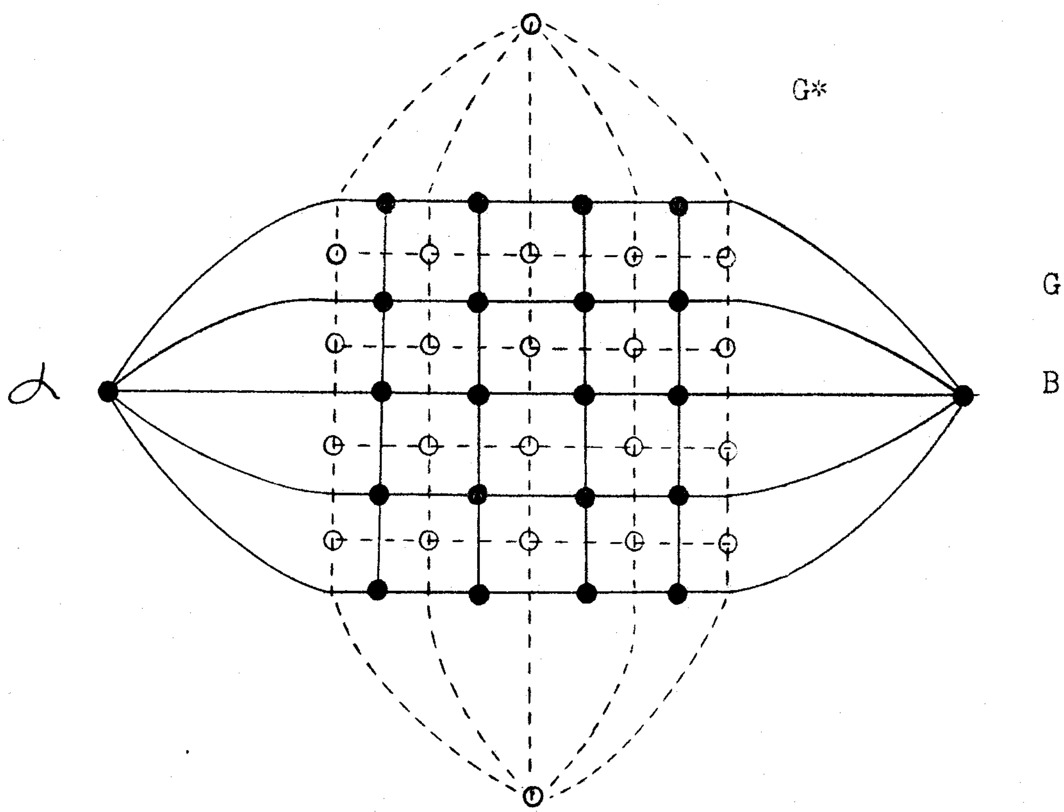

Fig. 3

A moves by "solidifying" the line between two vertices of the graph G. B moves by "blocking" such a link in the graph. A cannot solidify a blocked link and B cannot block a solidified link. Note that: blocking in G = solidifying in G* and v.v. Player A wins if he solidifies a path from α to B and B wins if he can block all paths from α to B. By considering the duality between G and G* one can see that the player who moves first can win if he plays properly.

These graph games are mechanized by letting each link originally be a resistor call of the same size). A solidified link is replaced by a straight connection (zero resistance) and a blocked link becomes a disconnection (∞ resistance). An electrical current flow is set up by imposing plus and minus potentials at α and B. When it is the machine's turn to move it moves at the link where the most current is flowing, regardless of whether its aim is to block or to solidify.

Shannon reports that in their experience the machine has never been beaten if given the first move. But its infallibility has not been proved.

The existence of electrical analogue strategies in these games suggests the possibility that such a procedure might profitably be applied to other games.

If a military engagement be regarded as a game, such an approach might be useful in suggesting overall strategy. Of course, it is not probable that any analog method quite as simple as those described above would be very useful for most military applications.

mb