Entrepreneurship as Recombinant Growth

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ABSTRACT. The paper presents a model of the entrepreneur as an undertaker of new combinations of ideas. Technology is seen as a set of ideas in a metric technology space where new knowledge is created by the combination of older ideas in the spirit of Schumpeter (1934), Weitzman (1998) and Olsson (2000). Given some intuitive assumptions, we demonstrate that technological progress generated by the convex combination of ideas is constrained by five factors: First, the combinatory process eventually leads to the exhaustion of technological opportunity. Second, the cost of combining ideas increases with the technological distance between the originating ideas. Third, profits are maximized when ideas are combined that are technologically close. Fourth, the technology set is constrained by a social possibility set of socially acceptable ideas. Fifth, the boundaries implied by the ruling technological paradigm limit the scope for eternal recombinant growth.

1. Introduction

It is often acknowledged that entrepreneurial activity is not easily included in standard economic models. The nature of the entrepreneur is in many ways at odds with the rational, well-informed capitalist in the theory of the firm and with the routinized, large-scale R&D processes modeled in endogenous growth theory. Nevertheless, technological creativity is generally regarded as a fundamental determinant of economic progress.[1] In this paper, we follow in the spirit of Schumpeter (1934), Weitzman (1998) and Olsson (2000) in viewing the entrepreneur as an undertaker of new combinations of ideas. In our formal model, binary pairings are carried out in a metric technology space where ideas are separated by technological distance. Convex combinations of ideas at the technological frontier lead to the expansion of the technology set. The process is constrained by (i) technological opportunity, (ii) the costs of combination, and (iii) the revenues of combination, (iv) the institutional framework, and (v) the ruling technological paradigm.

The notion of innovations as a result of new combinations is described in Schumpeter (1934). In Schumpeter's own words, "The carrying out of new combinations means, therefore, simply the different employment of the economic system's existing supplies of productive means . . ." (Schumpeter, 1934, p. 68). All economic development is the result of such new combinations. This combination process is very different from the dynamic analysis of orthodox microeconomic theory since the former involves ". . . spontaneous and discontinuous change . . ./and/disturbance of equilibrium, which forever alters and displaces the equilibrium state previously existing." (p. 64). Schumpeter distinguishes between five different kinds of new combinations: The introductions of (i) a new good, (ii) a new method of production, (iii) a new market, (iv) new source of supply of intermediate goods, and (v) a new organization. Any person who carries out new combinations is defined as an *entrepreneur*.

Weitzman's (1998) starting point is the production functions for new knowledge in modern growth theory. Like Schmookler (1966), Weitzman uses combinatorial mathematics to describe the

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process whereby older ideas are combined to create new ideas. A metaphor for this process is an agricultural research station that develops improved plant varieties by cross-pollinating existing plant varieties. The growth of knowledge depends on the number of "hybrid ideas" – combinations of ideas not previously combined – and on the amount of resources devoted to developing the hybrid ideas into usable form. When this kind of knowledge creation process is introduced into a growth framework, the ultimate limits to output growth do not lie in our ability to create new ideas, since the number of hybrid ideas eventually will become infinite. Rather, "recombinant growth" is limited by our ability to process these ideas into increased knowledge. The explosive behaviour over time of the stock of hybrid ideas might also ward off the diminishing returns of capital, implying the possibility of positive secular growth.

This paper follows Olsson (2000) in combining the Schumpeterian view of the entrepreneur as an agent carrying out new combinations, with Weitzman's idea-based theory of recombinant economic growth. It is inspired by results from the empirical patent literature (Jaffe, 1986; Griliches, 1992; Jaffe et al., 1993; Audretsch and Feldman, 1996). We present a formal model where the body of contemporary technology is regarded as a set in a metric "technology space". In this technology space, ideas are separated by "technological distance" and the ideas contained in the technology set form an infinite, bounded, closed and connected set. The entrepreneur, who expands the technology set by convex, binary combinations of existing ideas, plays the key role in this setup.

The set theoretic framework allows us to analyze five intuitive constraining factors to the process of recombinant growth that are not captured by Weitzman's (1998) model: (i) When all convex combinations have made, technological opportunity is exhausted. (ii) Costs of combining ideas increase with technological distance. (iii) All else equal, profits are maximized when the ideas combined are technologically close. (iv) Rational entrepreneurs will not create ideas that are not socially acceptable. (v) Technological paradigms constrain the possibilities for convex combinations, but paradigm shifts are more likely the nearer the exhaustion of technological opportunity.

Section 2 briefly presents Weitzman's recombinant growth model, based on combinatorics. Section 3 outlines this article's basic framework while Section 4 treats each of the five constraints in turn. Section 5 concludes the paper.

2. Recombinant growth

Recombinant growth as modeled by Weitzman (1998) uses combinatorial theory as its primary vehicle. Binary pairings of ideas are performed out of a finite set of A already existing ideas. The number of possible binary combinations at time t, $Z_2(A_t)$, is given by the formula

$$Z_2(A_t) = \frac{A_t!}{(A_t - 2)!2!} = \frac{A_t(A_t - 1)}{2}\,.$$

The growth of the number of ideas at time $t + 1$, ΔA_{t+1}, is given by

$$A_{t+1} - A_t = \Delta A_{t+1} = \min[\pi(Z_2(A_t) - Z_2(A_{t-1})), J]$$

where π is the probability of raising a new idea out of the newly combined "seed" ideas $(Z_2(A_t) - Z_2(A_{t-1}))$ and where J is the level of resources devoted to this combination process. The minimum of the number of successful new combinations $\pi(Z_2(A_t) - Z_2(A_{t-1}))$ and the level of resources (J) determines the growth of the stock of ideas. Given that $\pi(Z_2(A_t) - Z_2(A_{t-1})) < J$, it can be shown that the growth rate of ideas $\Delta A_{t+1}/A_t$ increases with the size of A_t. The capacity to process new ideas, on the other hand, increases linearly (J being a fraction of total output). Hence, the long-run constraint to knowledge growth does not lie in the combinatorial idea formation process but hinges on the resources devoted to cultivating the numerous ideas into useable form.

This article also embraces the notion of technological progress as arising from the combination of ideas. However, by integrating the main principle behind recombinant knowledge growth into a metric *technology space*, we show that some important constraints to knowledge growth appear within the combinatorial process itself. Unlike Weitzman's model, the framework of metric spaces allows us to analyze aspects like technological distance between ideas, areas of technological opportunity, the costs of combining ideas, and the constraining influence of existing societal institutions and technological paradigms. The basic structure follows Olsson (2000) and is intended to serve as a mathematical illustration of an innovating entrepreneur's behaviour rather than as an empirically testable model. We acknowledge of course that technological progress is far too complex to be fully grasped by a set of abstract mathematical relations.

3. The technology set

The set of technological knowledge at time t, henceforth referred to as A_t, is assumed to have the following characteristics:[2]

Assumption 1: $A_t \subset I \subset \mathbb{R}^k_+$, i.e. the technology set A_t is a subset of the *technology space* I which is defined in metric space $\mathbb{R}^k_+$, where k is the number of dimensions of I.

Technology space I is the universal set of all possible technological ideas in the past, in the present, and in the future. It is defined in a metric space $\mathbb{R}^k_+$. k stands for the dimensions of technological thinking; complexity, abstractness, aesthetics, etc. It may not be possible to determine how many such dimensions there are or indeed what exactly a dimension in technology space is. However, despite the obscurity of the concept, we believe that most people would agree that different dimensions really exist and that ideas can be categorized as being more or less strongly identified by these dimensions. For instance, in the empirical patent literature, the detailed classification of patents in some sense measures a patent's strength of identity in certain dimensions (Griliches, 1992).

The technology set A_t contains all the ideas about the production of goods and services, which are held for relevant at time t. It includes not only the latest inventions but also a vast amount of older technological ideas upon which today's inventions are built. Hence, ideas like "wheeled transport", "the combustion engine", and "the automobile" are all contained in A_t. Per definition, $\mathrm{I} = A_t \cup A_t^C$ where A_t^C is the complement of A_t, containing all ideas which are not a part of technological knowledge at time t. A_t^C therefore contains all future inventions that have still not been made. Note that the ideas in A_t are metaphysical *images*, or *reflections*, of physical goods and processes in the real world. There does not necessarily exist a perfect correspondence between the technology set and the set of real world objects at all t. Technological ideas are *disembodied* from the objects which they reflect and often exist independently from the physical world, for instance after a war when a great part of the physical objects have been demolished.

The purpose of introducing a metric idea space is to allow a discussion of the *technological distance* between ideas:

Assumption 2: If $i_1, i_2 \in I \subset \mathbb{R}^k_+$, then $d(i_1, i_2) \in \mathbb{R}_+$ is the *technological distance* between i_1 and i_2.

Technological distance reflects the notion that ideas are more or less related. For instance, the two ideas "steel" and "the Bessemer process" are more closely related than the ideas "the Bessemer process" and "the spinning wheel". In other words, the technological distance between the first two ideas is smaller than that between the latter two. Technological distance is thus the real-valued metric that defines ideas' relative position in technology space.[3] As we will demonstrate below, the technological distance between ideas should be of great importance for the potential of combining ideas.

A more technical assumption the following:

Assumption 3: The technology set A_t is *infinite*, *bounded*, *closed* and *connected* at all t.

The assumed infiniteness of the technology set implies that, unlike in Weitzman's model, it is meaningless to discuss the number of ideas since this number is always infinite. However, since the set is bounded, it is possible to analyze the "size" of the set. The size of any set $\Omega \subset I$ is given by the function $s(\Omega) \in \mathbb{R}_+$. A_t is bounded because we assume that there is a real number M and a point $i_l \in I$ such that the intellectual distance $d(i_l, i_m) < M$ for all $i_m \in A_t$. Boundedness means that there is a finite limit to the maximum intellectual distance between two ideas in the technology set. The set might therefore "grow" or "shrink" over time.

The closedness property indicates that A_t

contains all its boundary points. The set of A_t:s boundary points is defined as the intersection between the technology set and the limit points of its complement, $bdy(A_t) = A_t \cap l_p(A_t^C)$ where $l_p(A_t^C)$ equals A_t^C:s set of all limit points. These boundary points, which form the "surface" of the technology set, constitute *the technological frontier*. Since A_t is bounded and closed, it follows that it must also be compact.

Lastly, connectedness means that the technology set is always coherent and is not made up of separate islands in idea space. In formal terms, the connectedness of A_t implies that there are no two open subsets Ω_1, $\Omega_2 \subset I$ such that $\Omega_1 \cap \Omega_2 = \varnothing$, $A_t \cap \Omega_1 \neq \varnothing$, $A_t \cap \Omega_2 \neq \varnothing$ and $A_t \subset (\Omega_1 \cup \Omega_2)$. All knowledge is by nature cumulative, highly path dependent and new knowledge is created out of older knowledge (Dosi, 1988). These properties imply that the technology set can be described as a single coherent entity or body embedded in technology space, whose shape changes with technological progress.

The key assumption in this paper is that entrepreneurs create new technological ideas by combining older ideas. There is indeed a great amount of anecdotal evidence that one might advance in support of this axiom. In pre-historical times, archeological findings suggest that the two already familiar ideas "ceramics" and "wooden basket" were combined to create the invention of "pottery" (Diamond, 1997). One of the most important innovations during the later Middle Ages was the windmill. The windmill combined the ideas "watermill" and "sail" (Mokyr, 1990). Weitzman (1998) uses the example of Edison's invention of the electric light bulb. This invention combined the ideas "electricity" and "candle" to create the new idea "electric candle". Numerous similar examples can be found in almost any book on the history of technology.

Assumption 4: During normal technological advance, all new ideas i_n are the outcome of *convex combinations* of the type $i_n = \lambda_n i_l + (1 - \lambda_n) i_m$ of older ideas i_l, $i_m \in A_t$ with $\lambda_n \in (0, 1)$.

The new idea i_n thus ends up as a point somewhere on the line between i_l and i_m.

Our view of normal technological advance is inspired by Kuhn (1962). Kuhn defines normal science as the "mopping-up" or "puzzle-solving" activities that most scientists or entrepreneurs are engaged in all their lives. It involves an incremental growth of knowledge from which nothing radically new is learned. The type of convex combinations of ideas described in Assumption 4 is similar to what Kuhn refers to as normal science. Above all, it is our formalization of Schumpeter's (1934) notion of innovations as new combinations. Entrepreneurs utilize the nonconvexities in the technology set, which are therefore the areas of *entrepreneurial* or *technological opportunity*. The level of technological opportunity might vary considerably over time and between fields of research (Jaffe, 1986). The gradual process of convex combinations that expands the knowledge set by eliminating nonconvexities, is fundamentally different from the kind of combinations outlined by Weitzman (1998).

The nonconvexities of the technology set – or the areas of technological (entrepreneurial) opportunity – are defined as follows:

Assumption 5: The *technological opportunity set* B_t is the smallest set such that $A_t \cup B_t$ is convex. $A_t \cup B_t = P_t$ constitutes the *technological paradigm*.

This is shown in Figure 1 which, without loss of generality, assumes $k = 2$. The area between the technology set and the limiting lines is the smallest set that makes $A_t \cup B_t$ convex. It is the greatest possible expansion of A_t that can be made by convex combinations. This is therefore the technological opportunity set, the unique area into which entrepreneurs might expand existing technological knowledge. Its close relation to A_t implies that B_t is infinite and bounded, but unlike A_t, it is not necessarily connected since the areas of nonconvexity might be disjoint sets. Furthermore, we assume that B_t is open relative to A_t but closed relative to I.[4] Thus, $A_t \cup B_t = P_t$ is also closed relative to I.

The union of the technology set and the technological opportunity set is what we define as the technological paradigm. Since normal technological progress implies that increases in A_t result from exactly corresponding decreases in B_t,

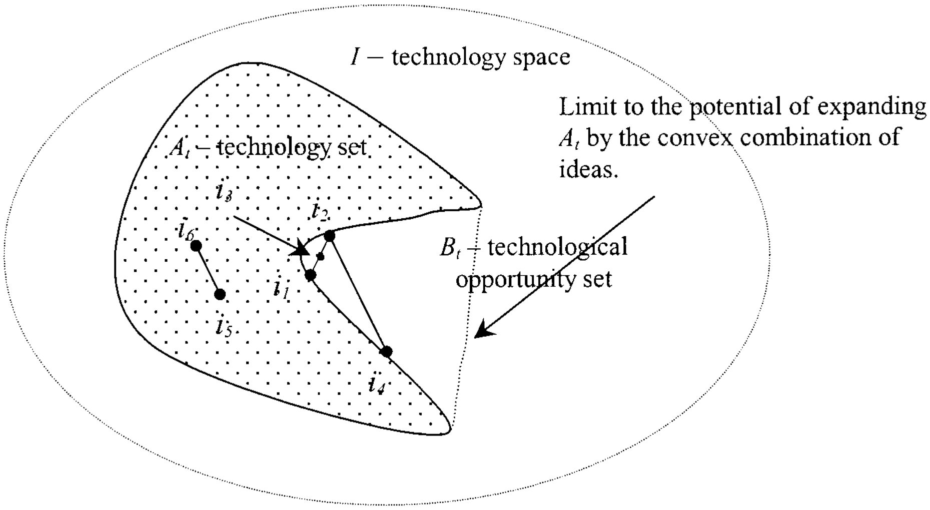

Figure 1. The technology set.

normal years will always be characterized by $A_t \cup B_t = P_t = A_{t+1} \cup B_{t+1} = P_{t+1}$. By the same logic, we are able to define a paradigm shift in the following manner:

Assumption 6: A technological paradigm shift has occurred at $t + 1$ if and only if $P_{t+1} \neq P_t$.

We believe these definitions of paradigms and paradigm shifts captures the essence of less formalized definitions in the literature. Dosi (1988, p. 1127) defines a technological paradigm as an "outlook", "a definition of the relevant problems", "a pattern of enquiry". At the core of the technological paradigm is usually a particular "general purpose technology" that can be used in many sectors of the economy (Helpman, 1998). There are several examples of such paradigms; agriculture, the printing press, the steam engine, the automobile, atomic power, to name a few. Paradigm changes, which are technological revolutions, put all older knowledge in a new light. It is characterized by the "swarm-like appearance" of entrepreneurial activity and new combinations (Schumpeter, 1934).

In our model, a paradigm shift is an indeterminate reconfiguration of P_t. Technology sets might grow or shrink unpredictably and technological distance between ideas and ideas' positions in technology space might change. The most important effect, however, is that a new paradigm reintroduces nonconvexities and entrepreneurial opportunity. In Section 4.5, we will present one possible mechanism of paradigm shifts.

4. Constraints to technological progress

The model outlined in Section 3 can be used to discuss five important constraints to technological progress not captured by Weitzman's recombinant growth model; (i) technological opportunity, (ii) costs of recombination, (iii) revenues of recombination, (iv) the impact of a social possibility set, and (v) technological paradigms.

4.1. *Technological opportunity*

Given the basic structure of our model, some intuitive propositions follow. The first one takes into consideration the potential of older ideas in the creation of new knowledge:

Proposition 1: Given any two technologically close ideas, it is only possible to expand the technology set by convex combinations of ideas on the part of the technological frontier that faces B_t.

Proof: Let $i_l, i_m \in A_t$ and let $d(i_l, i_m) \to 0$. New technological knowledge is created if

$\lambda_n i_l + (1 - \lambda_n) i_m = i_n \in B_t$ for any $\lambda_n \in (0, 1)$. A necessary but not sufficient condition for this to be possible is that $i_l, i_m \in (A_t \cap l_p(B_t))$ where $l_p(B_t)$ is the set of all limit points of B_t. In other words, the ideas to be combined must belong to the technological frontier on the nonconvex part of A_t. In all other cases, we must have (by the logic of convex sets) that $\lambda_n i_l + (1 - \lambda_n) i_m \in A_t$, which means that no new knowledge is created.

The result can also be straightforwardly obtained by viewing Figure 1. What it says is essentially that ideas in the interior and on the convex parts of the technological frontier can not be used for gaining new technological knowledge by combining technologically close ideas. Only the very small subset of ideas consisting of ideas on the boundaries of A_t and B_t, can be used. This makes intuitive sense. Combinations of closely related trivial ideas seldom produce any new knowledge. As illustrated in Figure 1, the combination of i_1 and i_2, both at the technological frontier, yields new knowledge (i_3) while a combination of two interior ideas like i_5 and i_6, does not.

However, this result applies with certainty only for ideas whose technological distance is (infinitesimally) small. An entrepreneur who combines interior ideas that are technologically very distant, might still end up with new knowledge. In Section 4.2, we will discuss why such combinations rarely take place.

Proposition 2: Technological opportunity is exhausted when the technology set is convex.

Proof: When A_t is convex, the smallest set B_t that satisfies the convexity of $A_t \cup B_t$ is simply $B_t = \emptyset$. Since B_t is empty, technological opportunity is exhausted.

This circumstance imposes an important constraint on entrepreneurial activity and recombinant growth. The scenario is depicted in Figure 1. When the limit on the right is reached, no further convex combinations of the type described in Assumption 4 will expand the set and the technological opportunity set will be empty. In this sense, normal technological progress has diminishing returns; as knowledge increases, the set tends towards convexity, and the technological frontier shrinks. During such times, there will be productivity slowdowns in aggregate production and economies will stagnate. In the language of growth theory, knowledge under a given paradigm will approach a steady state level of zero growth, just as is the case for capital. This would also contradict the common assumption of endogenous growth theory that knowledge is not subject to diminishing returns (see e.g. Romer (1996)).

In the long run, the only rescue for the entrepreneur from this dead end is a paradigm shift that reintroduces nonconvexities and entrepreneurial opportunities. We will return to this crucial issue below.

4.2. *Costs of recombination*

A second important constraint to recombinant growth is the costs of combination. Existing recombinant growth theory in Weitzman (1998) discusses the average costs for developing productive new ideas from the set of hybrid ideas. Various scenarios are analyzed, where costs range from zero to infinity, but no motivation is given for why costs should be high or low. We suggest that the costs of combining ideas should depend on two broad factors: (i) the costs of human and physical resources used in the combinatory process, and (ii) technological distance between the ideas involved.

Assumption 7: The cost of creating a new idea i_n by combining two ideas $i_l, i_m \in A_t$ is a function $C(i_n) = wJ + \delta \times d(i_l, i_n) \times d(i_m, i_n)$ where wJ is the fixed cost of human and physical resources and $\delta > 0$ is a parameter.

Physical resources J and their rate of compensation w reflect the same kind of resources as those used in Weitzman's combinatory process. Included in J are costs for research labour, scientific instruments, rents for buildings, etc. The novel aspect of the function above is that total costs also depend on technological distance between the new idea i_n and its originating ideas i_l and i_m. In order to relate the logic behind the multiplicative distance function above, some preliminary remarks must be made.

By the definition of metric spaces, we know that distance between the ideas to be linearly combined equals the sum of the distances between the new idea and the originating ideas; $d(i_l, i_m) = d(i_l, i_n) + d(i_m, i_n)$ where $d(i_l, i_n), d(i_m, i_n) > 0$. Further, the size of $d(i_l, i_n)$ and $d(i_m, i_n)$ depend on $\lambda_n \in (0, 1)$ (Assumption 4). λ_n determines the location of the new idea along the line between the two originating ideas, as shown in Figure 1. The specific functional form in Assumption 7, where the two distances are multiplied, has the following straightforward implications:

Proposition 3: (a) Costs of combination increase with distance between the ideas to be combined. (b) With a given distance $d(i_l, i_m)$ between the two ideas to be combined, costs are maximized when $d(i_l, i_n) = d(i_m, i_n) = 0.5 \times d(i_l, i_m)$.

Proof: (a) Since $d(i_l, i_m) = d(i_l, i_n) + d(i_m, i_n)$, we must have that an increase in $d(i_l, i_m)$ is associated with an increase in $d(i_l, i_n) \times d(i_m, i_n)$ and hence also in costs of combination. (b) $d(i_l, i_n) = d(i_l, i_m) - d(i_m, i_n)$ can be substituted into the cost function to receive $C(i_n) = wJ + \delta \times [d(i_l, i_m) - d(i_m, i_n)] \times d(i_m, i_n)$. Holding $d(i_l, i_m)$ fixed but allowing $d(i_m, i_n)$ to vary, the first-order condition for maximum is $\partial C(\bullet)/\partial d(i_m, i_n) = \delta \times [d(i_l, i_m) - 2d(i_m, i_n)^*] = 0$ and the second order condition $\partial^2 C(\bullet)/\partial d(i_m, i_n)^2 = -2\delta < 0$, as required. Rearranging terms, we receive $d(i_m, i_n)^* = d(i_l, i_m)/2$, i.e. the maximum costs arise when the new idea is located exactly halfway between the originating ideas.

An unproven but intuitive corollary of Proposition 3(b) is that costs at a given distance $d(i_l, i_m)$ are minimized when $d(i_m, i_n) \to 0$ and $d(i_m, i_n) \to d(i_l, i_m)$ or vice versa.[5]

The reasoning behind the assumption and proposition above is simple. Starting with Proposition 3(a), it basically suggests that in order to combine two ideas that are far apart intellectually, an entrepreneur must have a good grasp of two dissimilar fields. This usually requires a greater amount of time and effort than being an expert in just one field. Most likely, it also requires a more advanced methodology. As technological distance between two ideas to be combined increases, costs therefore increase. This is illustrated in Figure 1 where the costs of combining i_1 and i_2 should be lower than combining i_2 and i_4.

But even with a given distance between the originating ideas, costs will increase the further towards the "middle" ($d(i_l, i_m)/2$) the new idea is located.[6] The logic of this result is that it should be more difficult to make two ideas meet halfway than to make them meet in close proximity to one of the two original ideas. Meeting halfway means a significant departure from both previous ideas whereas a new idea close to one of the sources must be regarded as a less demanding and presumably less costly achievement.

4.3. *Revenues of recombination*

Costs of combination, however, might still be allowed to be high if the entrepreneurial revenue from that same undertaking is even higher. Hence, the kind of combinations that will be made hinges on the nature of the revenue function and on the expected value of the new ideas. Is the economic value of new ideas an increasing function of $d(i_l, i_m)$ or not? We suggest that in the short run, profits are probably independent of technological distance. Innovations that are the result of combinations of distant ideas usually give rise to radically new knowledge, but such new ideas are not often associated with immediate great financial revenues. The reason is usually that the innovation is too radical for society when it is presented or ahead of its time. Diamond (1997) gives a telling example of Edison's remarkable invention of the phonograph, which combined the distant ideas "record keeping" and "sound". The great inventor considered its primary use to be the recording of dying people's last words. However, neither in that use nor as a dictating machine, the phonograph was a commercial success. There was simply no demand. Only after about 20 years did Edison reluctantly admit that the main commercial use of his phonograph was to play and record music.

If it is true, as we assert, that revenues are independent of the technological distance between ideas whereas costs increase with distance, then these assumptions naturally lead to Proposition 4:

Proposition 4: If revenues are a non-increasing function of $d(i_l, i_m)$, then Assumption 7 implies that rational entrepreneurs will combine ideas that are technologically close.

Proof: The profit maximization problem for the entrepreneur is $\max_{d(l_l, l_m) > 0} \Pi = R - wJ - \delta \times [d(i_l, i_m) - d(i_m, i_n)] \times d(i_m, i_n)$ where Π is profit and R is revenue. The first-order condition is simply $\partial\Pi/\partial d(i_l, i_m) = -\delta \times d(i_m, i_n) < 0$, i.e. an entrepreneur should optimally try to minimize the distance between the originating ideas.

Hence, innovations should be based on existing ideas that are in technological proximity with each other. Furthermore, R&D should be carried out within relatively narrow fields. The proposition is indeed supported by some empirical results. In a study of patent citations in the U.S., Jaffe et al. (1993) found that 55–60 percent of the cited patents in patent applications were from the same primary patent class as the originating patent and that this rate appeared to have increased slightly from 1975 to 1980. Naturally, this circumstance is a severe constraint to major technological progress.

However, we do not have anything near conclusive evidence of the assumption that revenues are unaffected by technological distance. The determinants of innovation demand is certainly a complex issue and several factors probably contribute. In the long run, as the phonograph example shows, it might very well be the case that revenues are greater the more radical the new idea. The general nature of the reward structure is a fundamental determinant of entrepreneurial activity, as discussed extensively by Baumol (1990). As his analysis shows, it is far from true that entrepreneurship that advances technological knowledge always has been the most rewarding kind of entrepreneurship. Just as often, unproductive rent seeking entrepreneurship has proven to be more rewarding. The rewards to productive entrepreneurial activity depend to a great extent on the institutions of society. This is what we will turn to next.

4.4. *The social possibility set*

The fourth constraint is not often recognized in formal economic modelling, recombinant growth theory being no exception. It concerns the role of institutions in the idea combination process.[7] As a set theoretical metaphor for social institutions, let us imagine that there exists a *social possibility set* $S_t \subset I \subset \mathbb{R}^k_+$. The social possibility set defines the ideas and the kind of technology that, at time t, is deemed to be socially acceptable. The shape and size of this set is determined by a society's institutions. Individuals pursue their knowledge activities in a context that is collectively constrained by organized patterns of socially constructed norms and roles and socially prescribed and proscribed activities expected of occupants of such roles.[8] They thus involve informal institutions like the sense of ethics and morale, freedom of thought, ideological beliefs, religious commands, codes of conduct and social norms, but also formal institutions such as property rights, corporate law, and universities.

Our simple proposal is that the technology set tends to expand somewhere in the subset $B_t^S = B_t \cap S_t$ where B_t^S is the "institutionally adjusted" technological opportunity set. If a new combination is located outside of this set, a new kind of cost must be added to the cost function in Assumption 7; the cost of perseverance in spite of social disapproval.

Assumption 8: In the case of $i_n \notin B_t^S$, i.e. if the new idea created is not part of the institutionally adjusted technological opportunity set, then costs of combination are $\varepsilon \times d(S_t, i_n)$ where $\varepsilon > 0$ is a parameter and where $d(S_t, i_n)$ is the minimum technological distance between the social possibility set and the new idea.

Thus, combinations which yield knowledge that falls outside the social possibility set will induce an additional cost that increases with the distance between the new idea and the set of socially acceptable ideas. The intuition is that the costs for an entrepreneur of coping with social and institutional resistance should be greater the further "away" the new idea is from the social possibility set. Often, the advance of technology might temporarily surge ahead of the existing constraints imposed by social institutions. It is ex ante uncertain whether a new idea will be deemed socially acceptable or not. The costs arise once the general public has reasserted its position and has clearly established whether the new knowledge has transgressed the acceptable boundaries. If the institutions of a society are sufficiently strong, unacceptable new knowledge will not be developed further and only what is socially acceptable will be printed in textbooks and be remembered.

An obvious implication of Assumption 8 is:

Proposition 5: If revenues are a non-increasing function of $d(S_t, i_n)$, then Assumption 8 implies that rational entrepreneurs will try to create new ideas that are socially acceptable.

Proof: If entrepreneurial profit is given by $\Pi = R - wJ - \delta \times d(i_l, i_n) \times d(i_m, i_n) - \varepsilon \times d(S_t, i_n)$, then $\partial\Pi/\partial d(S_t, i_n) = -\varepsilon < 0$, i.e. an entrepreneur maximizes profits in the corner solution where $d(S_t, i_n) = 0$, implying that $i_n \in S_t$.

History is indeed full of examples of how social institutions have hindered the accumulation of knowledge. Partly as a consequence of the introduction of Confucianism as a state philosophy in China during the Sung dynasty, i.e. around 1300 AD (Rozman, 1993; Duara, 1988), education in scientific subjects was strongly discouraged and discontinued, and only a few centuries later, China's technological lead over the West was lost. The religious institutions of medieval Europe before the Renaissance defined a social possibility set that was too narrow to include many of the scientific findings of the ancient Greeks and Arabs (Reinert and Daastøl, 1997). In present times, certain areas of entrepreneurial activity are considered socially unacceptable or at least highly controversial, like genetic engineering. In the years ahead, the commercial exploitation of the mapping of the human genome will probably lead to a number of conflicts between established social institutions and profit seeking entrepreneurs.[9]

4.5. *Technological paradigms*

Regardless of the social possibility set and the costs and revenues of combination, the dismal result from Proposition 2 still applies: When normal, short-distance combinations have made the technology set convex, technological opportunity is exhausted and no further progress is possible. The only rescue is then a paradigm shift, i.e. a technological revolution that transforms $A_t \cup B_t = P_t$. What might induce such an event?

In this article, we will briefly mention a model of paradigm shifts, more elaborately developed in Olsson (2001), that follows in the spirit of Schumpeter (1934) and Mensch (1979). Schumpeter (1934) observed that radical "new combinations" of ideas tended to appear in swarms or clusters, thereby giving rise to long waves in economic development. Mensch (1979) developed this hypothesis empirically as well as theoretically and made the argument that waves of basic innovations tended to appear every fifty years or so in the downturn phase of the long wave.

The line of reasoning here is similar; the evolution of the technological paradigm P_t depends crucially on B_t, i.e. on technological opportunity. As B_t nears exhaustion, entrepreneurial profits decline and the piecemeal expansion of knowledge by short-distance convex combinations gets increasingly difficult. The opportunity cost of trying to make long-distance combinations and of exploring unknown territories of technology space then steadily diminishes. Eventually, a new wave of drastic innovations emerges that brings with it areas of fresh technological opportunity and introduces a paradigm shift. Once the new paradigm is in place and opportunities are ripe, the incremental, short-distance combinatorial process resumes.

In formal terms, we suggest the following:

Assumption 9: Let $s(B_t) \in \mathbb{R}_+$ be the size of the technological opportunity set. The probability of the event $P_{t+1} \neq P_t$ is a decreasing function G of B_t, $\text{Prob}(P_{t+1} \neq P_t) = G(B_t)$, such that $\lim_{s(B) \to 0} G(B_t) = 1$ and $\lim_{s(B) \to \infty} G(B_t) = 0$.

Hence, in contrast to the neoclassical model of long-run growth, development never reaches a completely stationary state where economies stop advancing in the absence of exogenous shocks. On the other hand, as discussed in Olsson (2001), if the extraction of technological opportunity is very slow, then paradigm shifts will be rare and technological progress will be painstakingly sluggish.

In the long run, technological paradigm shifts are also likely to affect social institutions. In the section above, we did not present any dynamic function for the set of socially acceptable ideas, S_t. Many factors are likely to influence the development of S_t. The only conjecture that we propose here is that when the technological paradigm changes, the social possibility set is bound to change too:

Assumption 10: If $P_{t+1} \neq P_t$, then $S_{t+1} \neq S_t$.

In other words, although normal progress does not necessarily have consequences for society at large and although institutions might or might not restrain normal innovative activity, we postulate that technological revolutions always affect social institutions. The invention of sedentary agriculture, based on domesticated plants and animals, allowed the formation of cities, states, hereditary rule, collective worship, and writing. The recognition that the sun, not the earth, was the center of our planetary system, certainly changed man's perception of his place in the universe and had a great impact on religion and the public view of science. The use of steam engines in land and sea-based transportation led to substantial reductions in the cost and time of travel and made the world smaller for millions of people. Even stronger forces of globalization are shaking societies in the wake of the present revolution in communications technology. In the future, the revolutionary advances within biotechnology will force legislators as well as the average man or woman on the street to come to grips with issues such as the desirability of human cloning or the removal of genes causing what some might regard as undesired human traits such as homosexuality or rebellious behaviour. Without doubt, these moral considerations will change social institutions and contribute to the recurring transformation of the set of social possibility.

5. Concluding remarks

Schumpeter's view of the entrepreneur as an undertaker of new combinations and Weitzman's theory of recombinant growth can be joined into a model of a multidimensional technology space, where the technology set is a set of ideas which, during periods of normal science, is expanded through a process of convex combinations of already existing ideas, carried out by entrepreneurs. Given the structure of our formal model, five intuitive constraints to recombinant growth appear: (i) When the knowledge set is convex, technological opportunity is exhausted. (ii) It is more costly to combine ideas which are technologically far apart. (iii) If revenues from idea combination are a non-increasing function of technological distance, rational entrepreneurs will always combine ideas that are technologically close. (iv) If revenues from recombinant growth are not greater if the new idea is outside the social possibility set of accepted ideas, then rational entrepreneurs will strive to create ideas that are socially acceptable. (v) Technological paradigms constrain recombinant growth, but paradigm shifts are more likely the smaller the technological opportunity set. Paradigm shifts lead to changes in the social possibility set.

As it stands, the model outlined in this article is a first step towards an idea-based formal theory of recombinant technological progress and entrepreneurial activity. It should be seen as an illustration of some possibly neglected aspects of knowledge formation that possibly can be analyzed more stringently within the framework presented here. We believe that the description of entrepreneurship as a process of recombinant growth is a potentially useful metaphor which might be further developed in a number of directions, theoretically as well as empirically.

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Notes

1 See for instance Audretsch (1995), Baumol (1990; 1993), Mokyr (1990), Murphy et al. (1991), Gifford (1998), Iyigun and Owen (1999), or Wennekers and Thurik (1999).

2 An elaborate description of the properties of a more broadly defined "knowledge set" is provided in Olsson (2000). See

also Kauffman et al. (2000) for a model of search on a technological landscape.

[3] Similar concepts have been prevalent elsewhere in the economics literature. In the empirical research on patents and patent citations, Jaffe (1986) measures the "technological proximity" of firms and patents in some "technology space". In reviewing the literature on innovations, Dosi (1988) refers to "technological distance" between innovations.

[4] If B_t was closed relative to A_t, it would contain all its boundary points. However, this is not possible since we have already assumed that A_t:s technological frontier is contained in A_t.

[5] Note that $\lambda_n \in (0, 1)$ means that border solutions are not allowed. A border solution would imply that the new idea had the same location as one of the old ideas.

[6] We are grateful to an anonymous referee for this suggestion.

[7] There are many contributions on the direct effects of social institutions on economic growth (North, 1990; Hall and Jones, 1999; and others), but fewer attempts have been made at explaining the role of social institutions for technological progress. Mokyr (1990) and Reinert and Daastøl (1997) are notable exceptions.

[8] See e.g. Goodin (1996) for a general perspective, and North (1990) for the economic approach.

[9] We acknowledge that social institutions also might have the capacity of *reducing* the cost of recombining ideas, even though we do not explicitly model such a mechanism. Theory and empirical research, as well as historical experiences (in particular the fall of the Soviet empire), indicate that the social institution of the *market* actively promotes the combination of ideas to new knowledge, while planning strongly hinders it (see e.g. Jones, 1981; Rosenberg and Birdzell, 1986; Bernholz et al., 1998). What we want to point out is that the growth of knowledge does not only depend on the possibilities to recombine known technologies, but is strongly determined by social factors, which should therefore be integrated into the analysis of growth. The social limits to growth have been noted by many authors, most prominently Hirsch (1976).

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